Function of several variables 多元函数(p868)

$z = f(x, y)$ Function of 2 variables

Domain	(x, y)
Range	$f(x, y)$
Independent variables	x, y
Dependent variables	z

$w = f(x, y, z)$ Function of 3 variables

Exploration: Without using a graphing utility, describe the graph of each function of two variables.

$z = x^2 + y^2$

$z = x + y$

$z = x^2 + y$

$z = \sqrt{x^2 + y^2}$

$z = \sqrt{1 - x^2 + y^2}$

If $Q(x) = 0$, $\frac{dy}{dx} + P(x)y = 0$ is called homogeneous differential equation

Partial Derivatives 偏微分(p890)

If $z = f(x, y)$ then the **first partial derivatives** of f with respect to x and y are the functions f_x and f_y defined by

$$\frac{\partial}{\partial x} f(x, y) = f_x(x, y) = z_x = \frac{\partial z}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

Partial derivative with respect to x

$$\frac{\partial}{\partial y} f(x, y) = f_y(x, y) = z_y = \frac{\partial z}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}$$

Partial derivative with respect to y

This definition indicates that if $z = f(x, y)$ then to find f_x you consider y constant and differentiate with respect to x. Similarly, to find f_y you consider x constant and differentiate with respect to y.

The first partials evaluated at the point (a, b) are denoted by

$$\left.\frac{\partial z}{\partial x}\right|_{(a,b)} = f_x(a, b) \quad and \quad \left.\frac{\partial z}{\partial y}\right|_{(a,b)} = f_y(a, b)$$

Example1: Find Partial Derivatives f_x and f_y for

$$f(x, y) = xe^{x^2 y}$$

And evaluate each at point $(1, \ln 2)$.

Solution

Partial derivative with respect to x

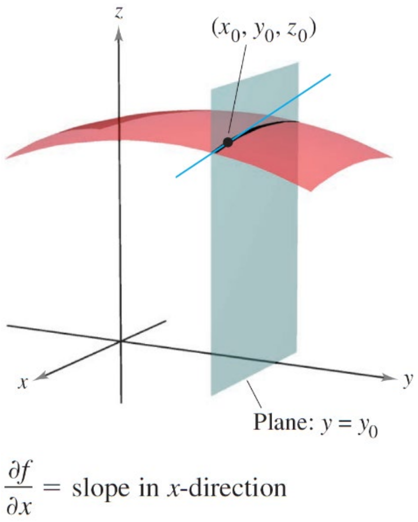
$f_x(x, y) = e^{x^2 y} + xe^{x^2 y}(2xy)$

$f_x(1, \ln 2) = e^{\ln 2} + e^{\ln 2}(2 \ln 2)$

Partial derivative with respect to y

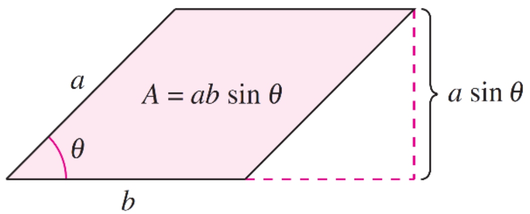
$f_y(x, y) = xe^{x^2 y}(x^2) = x^3 e^{x^2 y}$

$f_y(1, \ln 2) = e^{\ln 2} = 2$



The partial derivatives of a function of two variables, $z = f(x, y)$, have a useful geometric interpretation. If $y = y_0$ then $z = f(x, y_0)$ represents the curve formed by intersecting the surface $z = f(x, y)$ with the plane $y = y_0$ as shown in right Figure. Note that both the curve and the tangent line lie in the plane $y = y_0$.

Example2: Find rates of change. The area of a parallelogram with adjacent sides a and b and included angle θ is given by $A = ab \sin \theta$ as shown in the right Figure.



a. Find the rate of change of A with respect to a for a=10, b=20, and θ=π/6.

b. Find the rate of change of A with respect to θ for a=10, b=20, and θ=π/6.

Solution: (a) To find the rate of change of the area with respect to a hold b and θ constant and differentiate with respect to a to obtain

$$\frac{\partial A}{\partial a} = b\sin\theta$$

For $a=10$, $b=20$, and $\theta=\pi/6$.

$$\frac{\partial A}{\partial a} = b\sin\theta = 20\sin\frac{\pi}{6} = 10$$

(b) To find the rate of change of the area with respect to θ hold a and b constant and differentiate with respect to θ is

$$\frac{\partial A}{\partial \theta} = ab\cos\theta$$

For $a=10$, $b=20$, and $\theta=\pi/6$.

$$\frac{\partial A}{\partial \theta} = ab\cos\theta = 10\cdot 20\cos\frac{\pi}{6} = 100\sqrt{3}$$

Exercise: P896-1

1. $\frac{\partial}{\partial x}\left(\frac{x^2y}{y^2-3}\right)$

$$\frac{\partial}{\partial x}\left(\frac{x^2y}{y^2-3}\right) = 2x\left(\frac{y}{y^2-3}\right)$$

2. $\frac{\partial}{\partial y}\left(\frac{x^2y}{y^2-3}\right)$

$$\frac{\partial}{\partial y}\left(\frac{x^2y}{y^2-3}\right) = x^2\left[\frac{(y^2-3)-y(2y)}{(y^2-3)^2}\right] = x^2\left[\frac{-y^2-3}{(y^2-3)^2}\right]$$

9. $f(x,y) = x^2y^3$

$$\frac{\partial}{\partial x}(x^2y^3) = 2xy^3$$

$$\frac{\partial}{\partial y}(x^2y^3) = x^2 3y^2 = 3x^2y^2$$

12. $z = 2y^2\sqrt{x}$

$$\frac{\partial}{\partial x}(2y^2\sqrt{x}) = 2y^2\left(\frac{1}{2}\right)x^{-1/2} = y^2/\sqrt{x}$$

$$\frac{\partial}{\partial y}(2y^2\sqrt{x}) = 2\sqrt{x}(2y) = 4y\sqrt{x}$$

17. $z = x^2e^{2y}$

$$\frac{\partial}{\partial x}(x^2e^{2y}) = 2xe^{2y}$$

$$\frac{\partial}{\partial y}(x^2e^{2y}) = x^2e^{2y}(2) = 2x^2e^{2y}$$

18. $z = ye^{y/x}$

$$\frac{\partial}{\partial x}(ye^{y/x}) = ye^{\frac{y}{x}}(y)(-1)x^{-2} = -y^2e^{\frac{y}{x}}/x^2$$

$$\frac{\partial}{\partial y}(ye^{y/x}) = e^{y/x} + ye^{\frac{y}{x}}\left(\frac{1}{x}\right) = e^{y/x} + \frac{y}{x}e^{\frac{y}{x}}$$

19. $z = \ln x/y$

$$\frac{\partial}{\partial x}(\ln x/y) = \frac{1}{x/y}\left(\frac{1}{y}\right) = 1/x$$

$$\frac{\partial}{\partial y}(\ln x/y) = \frac{1}{x/y}(x)(-1)(y^{-2}) = -\frac{1}{y}$$

20. $z = \ln\sqrt{xy}$

$$\frac{\partial}{\partial x}(\ln\sqrt{xy}) = \frac{1}{\sqrt{xy}}\frac{1}{2}(xy)^{-1/2}y = \frac{1}{2x}$$

$$\frac{\partial}{\partial y}(\ln\sqrt{xy}) = \frac{1}{2y}$$

29. $z = \cos xy$

$$\frac{\partial}{\partial x}(\cos xy) = -\sin xy\,(y) = -y\sin xy$$

$$\frac{\partial}{\partial y}(\cos xy) = -x\sin xy$$

30. $z = \sin(x+2y)$

$$\frac{\partial}{\partial x}(\sin(x+2y)) = \cos(x+2y)$$

$$\frac{\partial}{\partial y}(\sin(x+2y)) = 2\cos(x+2y)$$